Polarization switching in a planar optical waveguide

V. Boucher, H. Leblond,* and X. Nguyen-Phu
Laboratoire POMA, UMR CNRS 6136, Université d'Angers, 2 Boulevard Lavoisier, 49045 Angers Cedex, France
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The multiscale expansion formalism is applied to the study of nonlinear planar optical waveguides. It allows us to describe the linear and nonlinear propagation for both transverse electric and transverse magnetic modes, and the interaction between them. An accurate computation of the nonlinear self- and cross-phase modulation coefficients allows one to give account of the polarization switching which has been observed experimentally.

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I. INTRODUCTION

Nonlinear optical guided modes and solitons in waveguides are of great interest, owing to their potential applications to optical signal processing devices [1–3], but they can hardly be described in a very rigorous way. The variational procedure proved to be a powerful tool for finding simple approximations of the optical waveguided modes [4,5]. A variety of numerical approaches has also been used [6–10]. Special interest has been given to analytical methods of finding solutions in slab waveguides [11–16] and of analyzing the important question of their stability [17–19]. In this paper we apply the multiscale expansion formalism to planar waveguides, in order to study the nonlinear propagation of a short localized wave packet, and the interaction between different guided modes. This formalism has already been applied to many problems. It was first used by Taniuti and Washimi in plasma physics [20], and recently in other domains of nonlinear physics, such as hydrodynamics or the physics of waves in ferromagnetic media [21]. However, there are only few applications of this formalism in its rigorous formal form to the study of waveguides [22,23]. The second of these papers is a preliminary to the present study. It discusses the propagation of a short pulse in a planar optical waveguide filled with a Kerr medium. The first order in the multiscale expansion gives the expressions of the linear modes. At a higher order, a nonlinear evolution equation is obtained. It takes into account the waveguiding structure, i.e., the boundary conditions, the waveguide parameters, and the nonlinear susceptibility of the medium. On the other hand, experiments have been performed in the same laboratory [24]. Once the formation of spatial solitons has been obtained, polarization measurements have been performed. They brought forward an instability of the transverse magnetic (TM) modes. Indeed, the polarization switches to a transverse electric (TE) one as soon as the input beam has a nonzero component in the corresponding direction. The aim of the present paper is to give a theoretical account of this observation, through a rigorous derivation of the model equations governing the nonlinear interaction between the modes.

*Electronic address: herve.leblond@univ-angers.fr

II. DERIVATION OF A MODEL

A. Setup and scales

We consider a wave propagating in a Kerr-like nonlinear dielectric waveguide constituted by a nonlinear film bounded by two linear media (Fig. 1). The essential physical assumptions read as follows. First, the amplitude of the wave is small, the approximation is weakly nonlinear. This smallness is purely formal, because the description of nonlinearity is rather phenomenological, a theoretical reference value can be found only through the quantum theory. Second, the pulse length L is very large with regard to the wavelength λ, and very small with regard to the propagation length D. Thus, three length and time scales are involved, $\lambda, L\approx\lambda/\varepsilon$, $D\approx\lambda/\varepsilon^2$.

Assuming the medium to be nonmagnetic, the Maxwell equations reduce to the following wave equation for the electric field $\mathbf{E}$:

$$\Delta\mathbf{E}-\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E})=\frac{1}{c^2}\partial_t^2[\mathbf{E}+\mathbf{P}], \tag{1}$$

where c is the light velocity in vacuum. The time variable t is rescaled as $t'=ct$, so that c takes the value 1. The primes are omitted below. We denote by ∂_t the derivative operator $\partial/\partial t$ with regard to the time variable t and so on. $\boldsymbol{\nabla}$ is the three-dimensional gradient operator relative to the space variables x, y, and z. We assume that $\mathbf{P}$ can be described by the following standard model: It splits into the sum $\mathbf{P}=\mathbf{P}_L+\mathbf{P}_{NL}$ of a linear part $\mathbf{P}_L$ satisfying

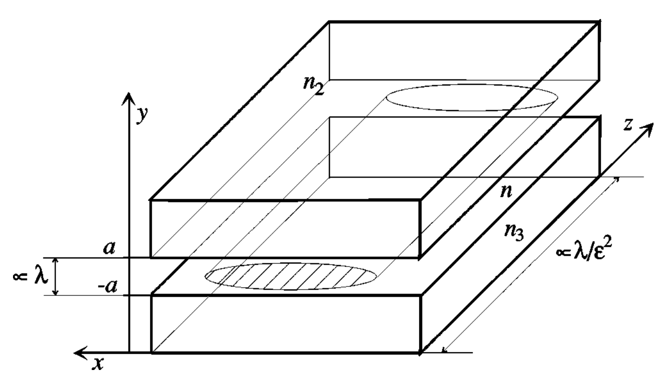

FIG. 1. Space scales involved in waveguide geometry.

$$\mathbf{P}_L = \chi^{(1)} * \mathbf{E} = \int_{-\infty}^{t} dt_1 \chi^{(1)}(t-t_1)\mathbf{E}(t_1), \tag{2}$$

and a nonlinear part $\mathbf{P}_{NL}$ corresponding to the nonlinear polarization such that

$$\mathbf{P}_{NL} = \chi^{(3)} * (\mathbf{E},\mathbf{E},\mathbf{E}) = \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \int_{-\infty}^{t_2} dt_3 \chi^{(3)}(t-t_1, t-t_2, t-t_3) : \mathbf{E}(t_1)\mathbf{E}(t_2)\mathbf{E}(t_3). \tag{3}$$

$\chi^{(1)}$ and $\chi^{(3)}$ are, respectively, the linear and third-order nonlinear susceptibility tensors. Experiments have been performed using a liquid medium such as CS_2 [24]. Then, as in any centrosymmetrical material, the second-order nonlinear susceptibility tensor $\chi^{(2)}$ is zero. This condition is assumed to be satisfied here. The fields **E** and **P** undergo a threefold expansion: in a power series of some small parameter ε, in a series of harmonics of each fundamental frequency, and on the linear modes of the waveguide.

This expansion is written as

$$\mathbf{E} = \sum_{l,j} \varepsilon^l \mathbf{E}_l^j e^{i\varphi_j}, \tag{4}$$

where the index j labels both the phase φ_j and the corresponding amplitude E_l^j. It refers either to a waveguide mode or to one of its harmonics. The amplitudes $\mathbf{E}_l^j$ are functions of y and of slow variables to be specified. The polarization density **P** is expanded in the same way.

B. Study of an isolated mode

In a preliminary study [23], we considered the propagation of a wide enough short pulse, of the "temporal" type, corresponding to a unique mode of the guide. We performed a standard nonlinear Schrödinger (NLS)-type expansion, which used the slow variables

$$\tau = \varepsilon\left(t - \frac{z}{v}\right), \quad \zeta = \varepsilon^2 z. \tag{5}$$

The y dependency describes the transverse structure of the waveguide modes. y is a variable of order ε^0, not a slow variable, which gives an account of the following assumption: the waveguide thickness has the same order of magnitude as the wavelength λ. The variable τ describes the longitudinal or temporal shape of the pulse, in a frame moving at the wave packet velocity v. It is a slow variable of order ε, which means that the pulse length has the same order of magnitude as $L = \lambda/\varepsilon$. The variable $\xi = \varepsilon x$ gives account of the transverse shape of the beam, in the same scale as τ, while ζ is the variable that describes the evolution of the pulse shape during the propagation. Its order is ε^2, giving account for propagation distances about $D = \lambda/\varepsilon^2$.

This scaling is almost the same as is commonly used for the derivation of the nonlinear Schrödinger equation in bulk media [25]. The main difference lies in the existence of the transverse variable y. It can thus describe a beam of some hundred micrometers width, propagating in a waveguide a few micrometers thick. For instance, the wavelength λ being about 0.5 μm, and the perturbative parameter $\varepsilon \simeq 5\times 10^{-3}$, the order of magnitude of the pulse width and length is $L = \lambda/\varepsilon \simeq 100$ μm, which corresponds to a characteristic duration of 0.3 ps, and the propagation distance is about $D = \lambda/\varepsilon^2 \simeq 2$ cm.

The nonlinear term, being cubic, appears directly in the nonlinear propagation equation, and does not give rise to any particular problem. The difficulty comes from the linear part: description of the guiding and treatment of the wave packet. The boundary conditions at the two interfaces are that of the electromagnetism, and the continuity conditions for the magnetic field must be taken into account. We retrieve in this way the complete description of the guided modes, TE, and TMs. In the TM modes, the electric field **E** has a longitudinal component. After a long computation, we derive the NLS equation that describes the evolution of the amplitude of a temporal pulse for an isolated mode, submitted to its self-phase modulation only, as

$$2ik\partial_\zeta A - kk''\partial_\tau^2 A + \Gamma|A|^2 A = 0, \tag{6}$$

where A is the wave amplitude. In this way we obtain in particular an explicit expression of the self-phase modulation coefficient Γ which takes into account the waveguide structure. For the TE modes, this expression is [23]

$$\Gamma = \Gamma_e = \gamma_1 \hat{\chi}^{(3)}_{xxxx}(\omega,\omega,-\omega), \tag{7}$$

where the coefficient γ_1 has a complicated expression that involves the guide thickness a, the transverse wave vector q of the guided wave, and the decay lengths $1/k_2$ and $1/k_3$ of the two evanescent waves that exist on each side of the guiding layer. Notice that q, k_2, and k_3 depend on the mode, and through it on the parameters of the guide: its thickness a and the optical indices n, n_2, and n_3 of the three media. The expression of the nonlinear coefficient Γ for a TM mode is

$$\Gamma = \Gamma_m = \eta_{1a}\hat{\chi}^{(3)}_{xyxy}(\omega,\omega,-\omega) + \eta_{1b}\hat{\chi}^{(3)}_{xyyx}(\omega,\omega,-\omega). \tag{8}$$

The coefficients η_{1a} and η_{1b} are expressed also as functions of these parameters. A plot of the values of Γ_e and Γ_m corresponding to the first two guided modes TE_1 and TM_1, for values of the parameters close to that of the experiments, is given in Fig. 2. The difference between these two quantities is weak, but can explain the observed polarization instability.

III. INTERACTION BETWEEN TWO MODES

A. The physical situation

While the previous ansatz was defined in order to be as close as possible to the standard derivation of a NLS equation as in Ref. [26], we intend here to set the problem close to some experimental situation, which is as follows: The pulse length is about 30 ps, which is so long with regard to the optical period that the longitudinal variations of the pulse should be neglected. The value of the group velocity disper-

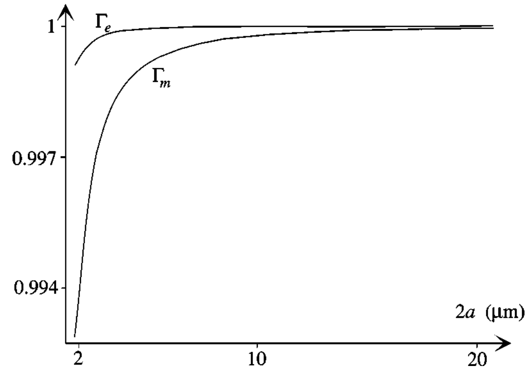

FIG. 2. Normalized nonlinear coefficients Γ_e and Γ_m for the TE_1 and TM_1 modes, respectively, vs the waveguide thickness $2a$.

sion k'' in the considered materials is very small, which improves this approximation. Thus, formally, we consider a stationary beam, not modulated in time. The amplitudes $\mathbf{E}_l^j$ are functions of y and of slow variables (ξ,ζ) defined by

$$\xi=\varepsilon x, \quad \zeta=\varepsilon^2 z. \tag{9}$$

We are interested in soliton-type propagation, often described as a balance between diffraction and nonlinearity. This effect can arise only if the propagation length has the same order of magnitude as the diffraction length or Rayleigh length, namely, the latter is $L_R=kw^2$ where w is the beam waist. In the present case, the beam waist is about a few $L=\lambda/\varepsilon$. The Rayleigh length associated with L is then $L_R=k\lambda^2/\varepsilon^2$. This shows that the correct order of magnitude of the variable ζ is ε^2.

Second, the wave field decomposes on the linear propagation modes of the waveguide. There are two classes of waveguide modes, TE (TE_1, TE_2, . . .) and TM (TM_1, TM_2, . . .), corresponding to the two perpendicular transverse linear polarizations. Interaction between modes is most efficient when their group and phase velocities are very close together. It can be seen that the propagation constants of a TE_n and a TM_n mode with the same n are very close together, while the difference between the propagation constants for a TE_n and a $TE_{n'}$ mode with $n\neq n'$ is much larger (Fig. 3). Therefore the study of nonlinear interaction between modes is of particular interest when the modes considered are a TE_n mode and a TM_n one, with the same n. The beam used experimentally has essentially a Gaussian shape. The linear decomposition of such a Gaussian beam on the basis of the waveguide modes shows that the first modes take the major part of the energy as shown in Table I. Therefore we restrict the study to the case where the dominant term in expansion (4) of the electric field $\mathbf{E}$ involves only two modes, a TE one and a TM one, intended to be TE_1 and TM_1, despite this specification not being involved in the formal computation.

Formally, the dominant term of order ε^1 in Eq. (4) contains two fundamental phases $j=e$ or m with

$$\varphi_e=k_e z-\omega t, \quad \varphi_m=k_m z-\omega t, \tag{10}$$

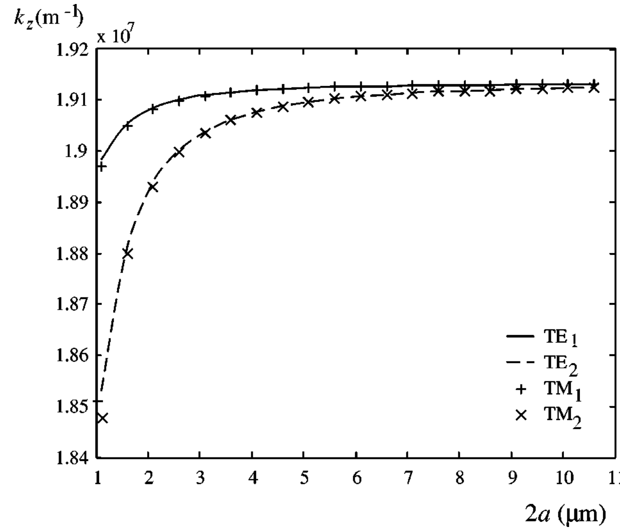

FIG. 3. Representation of the wave vector k computed for the TE_1, TE_2, TM_1, and TM_2 modes, against the guide thickness $2a$. The difference between the values of k obtained for a TE and for the relative TM mode is very small, even in very thin guides.

and the conjugated phases $\varphi_{-e}=-\varphi_e$ and $\varphi_{-m}=-\varphi_m$. At a higher order, the phase of the mode harmonic j has the following expression:

$$\varphi_j=k_j z-\omega_j t=p_e\varphi_e+p_m\varphi_m, \tag{11}$$

where p_e and p_m are arbitrary algebraic integers.

B. The "exact" computation

Expansions (4), together with definition (9) of the slow variables ξ and ζ, are substituted into the basic equations (1)–(3). Then the coefficients of each power of ε are collected to obtain a set of equations, which we solve order by order.

At first order, assuming that the amplitude $\mathbf{E}_1^e$ depicts a TE mode, we have

$$\mathbf{E}_1^e=\begin{pmatrix} E_1^{e,x} \\ 0 \\ 0 \end{pmatrix}, \tag{12}$$

where the amplitude $E_1^{e,x}$ satisfies

$$\partial_y^2 E_1^{e,x}+(\omega^2 n_i^2-k_e^2)E_1^{e,x}=0. \tag{13}$$

TABLE I. Decomposition of the incident Gaussian beam on the waveguide modes. For each $i=1,2,\ldots,6$, $I_i=\int_{-\infty}^{+\infty}G(y)E_i(y)dy$, where E_i and G are the transverse profiles of the TE_i mode and Gaussian input, respectively.

Mode	TE_1	TE_2	TE_3	TE_4	TE_5	TE_6
I_i/I_1	1	0.065	0.120	0.058	0.076	0.022

The resolution of Eqs. (13) with the electromagnetic boundary conditions leads to the dispersion relation for the TE waveguide modes [25]:

$$\tan(2q_e a) = \frac{q_e(k_{2e}+k_{3e})}{q_e^2 - k_{2e}k_{3e}}, \tag{14}$$

where q_j and k_{ij} ($j=e,m,\ i=2,3$) are the transverse wave vectors, related to the longitudinal wave vector k_j and the pulsation ω through the dispersion relation of optical waves in bulk media, which reads as ($i=2,3$ and $j=e,m$)

$$q_j = \sqrt{\omega^2 n^2 - k_j^2}, \tag{15}$$

$$k_{ij} = \sqrt{k_j^2 - \omega^2 n_i^2}. \tag{16}$$

In the same way, assuming that $\mathbf{E}_1^m$ depicts a TM mode,

$$\mathbf{E}_1^m = \begin{pmatrix} 0 \\ E_1^{m,y} \\ E_1^{m,z} \end{pmatrix}, \tag{17}$$

with

$$\partial_y E_1^{m,y} + i k_m E_1^{m,z} = 0, \tag{18a}$$

$$\partial_y^2 E_1^{m,z} + (\omega^2 n_i^2 - k_m^2) E_1^{m,z} = 0. \tag{18b}$$

The resolution of Eqs. (18), taking into account the electromagnetic boundary conditions, leads to the dispersion relation of the TM modes:

$$\tan(2q_m a) = \frac{q_m n^2 (k_{2m} n_3^2 + k_{3m} n_2^2)}{(q_m^2 n_2^2 n_3^2 - k_{2m} k_{3m} n^4)}. \tag{19}$$

The second order leads to equations analogous to Eqs. (13) and (18), with the index 1 replaced by 2.

The general expression of the nonlinear polarization vector which arises from the third order is

$$\mathbf{P}_3^j = \sum_{\varphi_{j_1}+\varphi_{j_2}+\varphi_{j_3}=\varphi_j} \hat{\chi}^{(3)}(\omega_{j_1},\omega_{j_2},\omega_{j_3}) : \mathbf{E}_1^{j_1}\mathbf{E}_1^{j_2}\mathbf{E}_1^{j_3}. \tag{20}$$

This polarization vector gives account of an eventual coupling between waveguide modes. For instance, regarding the $P_3^{e,x}$ component, apart from the permutations, there are two triplets that satisfy the phase matching conditions:

$$(\varphi_{j_1},\varphi_{j_2},\varphi_{j_3}) = (\varphi_e, \varphi_e, -\varphi_e), \tag{21a}$$

$$(\varphi_{j_1},\varphi_{j_2},\varphi_{j_3}) = (\varphi_e, \varphi_m, -\varphi_m). \tag{21b}$$

This way we obtain the following model:

$$2ik_e \partial_\zeta A_e + \partial_\xi^2 A_e + \Gamma_e |A_e|^2 A_e + g_2 |A_m|^2 A_e = 0, \tag{22}$$

$$2ik_m \partial_\zeta A_m + \partial_\xi^2 A_m + \Gamma_m |A_m|^2 A_m + h_2 |A_e|^2 A_m = 0. \tag{23}$$

Here A_e and A_m are the respective amplitudes of the two considered modes. The cross-phase modulation coefficients g_2 and h_2 are, as the self-phase modulation coefficients Γ_e and Γ_m, proportional to some components of the $\hat{\chi}^{(3)}$ tensor. The proportionality constants depend in a complicated, but explicitly known, way on the waveguide parameters.

The model [Eqs. (22) and (23)] gives account of self- and cross-phase modulation. It does not allow any energy exchange between the two modes. This feature is easily proven by multiplying the first equation by A_e^*, the second by A_m^*, subtracting the complex conjugate, and integrating over all values of ξ. This model assumes that the phase mismatch $(k_m - k_e)$ between the two modes is large, and is adapted to the description of the interaction between TE_n and $\mathrm{TM}_{n'}$ with $n \neq n'$. It obviously generalizes to two modes TE_n and $\mathrm{TE}_{n'}$ or TM_n and $\mathrm{TM}_{n'}$ modes with $n \neq n'$.

C. Four-wave-mixing term

However experiments show that an energy exchange can occur [24]. In fact, although formally distinct, the phase velocities of the two modes are very close together: their relative difference is about 10^{-6}. For instance, for a guide of 6 μm thickness, the relative difference $\delta k/k$ between the wave vectors of TE_1 and TM_1 modes is $1,1\times 10^{-6}$. When a finite value of the perturbative parameter ε is chosen, it is typically $\varepsilon = 10^{-3}$: a difference as small as this value of $\delta k/k$ must be considered as zero. From the mathematical point of view, $\delta k/k$ is finite, while ε tends to 0, which assumes that ε is much smaller than $\delta k/k$. The rigorous mathematical derivation of the system of Eqs. (22) and (23) becomes erroneous when we replace the infinitely small ε by some finite value. In particular, the system of Eqs. (22) and (23) is valid only when the relative difference between the wave vectors of the two modes is above the value of ε, which is by no means the case in the experimental situation considered. Therefore we use from now on a little more phenomenological approach. Formally, we write

$$k_e - k_m = \varepsilon^2 \delta k, \tag{24}$$

which corresponds to the physical orders of magnitude. It must be noticed that Eq. (24) is not perfectly consistent from the mathematical point of view, since initially the difference $k_e - k_m$ did not depend on ε. In fact it seems to be impossible to give account of the smallness of this quantity in the frame of the multiscale expansion in a perfectly rigorous way. Using Eq. (24) and writing down explicitly the phase factors $e^{ik_e z}$ and $e^{ik_m z}$ in expression (20) for the polarization vector components $P_3^{e,x}$, $P_3^{e,y}$, and $P_3^{e,z}$ yields, taking into account the $\hat{\chi}^{(3)}$ tensor symmetries,

$$P_3^{e,x} e^{ik_e z} = [3\hat{\chi}^{(3)}_{xxxx} |E_1^{e,x}|^2 E_1^{e,x} + 6\hat{\chi}^{(3)}_{xxyy}(|E_1^{m,y}|^2 + |E_1^{m,z}|^2) E_1^{e,x}] e^{ik_e z} + 3\hat{\chi}^{(3)}_{xyyx}[(E_1^{m,y})^2 + (E_1^{m,z})^2] E_1^{e,x*} e^{i(2k_m - k_e)z}. \tag{25}$$

(* denotes complex conjugation.) The phase mismatch $k_e - k_m$ can be written as

$$e^{i(2\varphi_m-\varphi_e)}=e^{i(2k_m-k_e)z-\omega t}=e^{i\varphi_e}e^{-2i\delta k\zeta}, \tag{26}$$

and the nonlinear polarization component $P_3^{e,x}$ becomes

$$P_3^{e,x}=3\hat{\chi}^{(3)}_{xxxx}|E_1^{e,x}|^2E_1^{e,x}+6\hat{\chi}^{(3)}_{xxyy}(|E_1^{m,y}|^2+|E_1^{m,z}|^2)E_1^{e,x}+3\hat{\chi}^{(3)}_{xyyx}[(E_1^{m,y})^2+(E_1^{m,z})^2]E_1^{e,x*}e^{-i2\delta k\zeta}. \tag{27}$$

Using the same procedure we get, for $P_3^{m,y}$ and $P_3^{m,z}$,

$$P_3^{m,y}=3\hat{\chi}^{(3)}_{xxxx}|E_1^{m,y}|^2E_1^{m,y}+6\hat{\chi}^{(3)}_{xxyy}(|E_1^{m,z}|^2+|E_1^{e,x}|^2)E_1^{m,y}+3\hat{\chi}^{(3)}_{xyyx}[(E_1^{m,z})^2+(E_1^{e,x})^2]E_1^{m,y*}e^{i2\delta k\zeta}, \tag{28}$$

$$P_3^{m,z}=3\hat{\chi}^{(3)}_{xxxx}|E_1^{m,z}|^2E_1^{m,z}+6\hat{\chi}^{(3)}_{xxyy}(|E_1^{m,y}|^2+|E_1^{e,x}|^2)E_1^{m,z}+3\hat{\chi}^{(3)}_{xyyx}[(E_1^{m,y})^2+(E_1^{e,x})^2]E_1^{m,z*}e^{i2\delta k\zeta}. \tag{29}$$

Formally, the phase matching is realized, in what concerns the z dependency, since it has been possible to incorporate the phase mismatch into a dependency with regard to the slow variable ζ.

The third order of the expansion leads to the equations

$$\partial_y^2E_3^{e,x}+(\omega^2n_i^2-k_e^2)E_3^{e,x}=-2ik_e\partial_\zeta E_1^{e,x}-\partial_\xi^2E_1^{e,x}-\omega^2P_3^{e,x}, \tag{30}$$

$$\partial_yE_3^{m,y}+ik_mE_3^{m,z}=-\partial_\zeta E_1^{m,z}-\frac{ik_m}{n_i^2}P_3^{m,z}-\frac{1}{n_i^2}\partial_yP_3^{m,y}, \tag{31}$$

$$\partial_y^2E_3^{m,z}+(\omega^2n_i^2-k_m^2)E_3^{m,z}=-2ik_m\partial_\zeta E_1^{m,z}-\partial_\xi^2E_1^{m,z}-\frac{ik_m}{n_i^2}\partial_yP_3^{m,y}-\frac{q_m^2}{n_i^2}P_3^{m,z}. \tag{32}$$

We solve the above differential equations where the right-hand-side member depends only on expressions calculated at the first order. Applying boundary conditions, it yields after a large amount of calculations the following nonlinear coupled propagation equations:

$$2ik_e\partial_\zeta A_e+\partial_\xi^2A_e+\Gamma_e|A_e|^2A_e+g_2|A_m|^2A_e+g_3A_m^2A_e^*e^{-2i\delta k\zeta}=0, \tag{33}$$

$$2ik_m\partial_\zeta A_m+\partial_\xi^2A_m+\Gamma_m|A_m|^2A_m+h_2|A_e|^2A_m+h_3A_e^2A_m^*e^{2i\delta k\zeta}=0. \tag{34}$$

The nonlinear interaction coefficients g_2,g_3,h_2,h_3 are

$$g_2=\gamma_2\hat{\chi}^{(3)}_{xyxy},\quad g_3=\gamma_3\hat{\chi}^{(3)}_{xyyx},$$

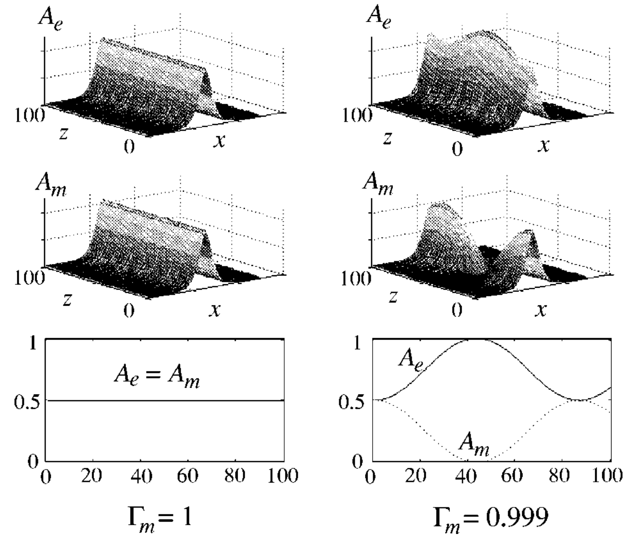

FIG. 4. Comparison of the evolution of the amplitudes of the two polarization components, for a small variation of the self-phase modulation coefficient Γ_m. On the left: evolution of the amplitudes A_e and A_m of the TE and TM modes, respectively, against transversal space x and propagation distance z, assuming the bulk values of the nonlinear coefficients. Bottom: evolution of the maximum value of the amplitudes. On the right: the same quantities but the nonlinear self-phase modulation of the TM mode Γ_m has been reduced from 1 to 0.999. Polarization switching is observed.

$$h_2=\eta_2\hat{\chi}^{(3)}_{xyxy},\quad h_3=\eta_3\hat{\chi}^{(3)}_{xyyx}, \tag{35}$$

where the coefficients γ_2, γ_3, η_2, and η_3 are expressed explicitly in terms of the waveguide parameters. They are given in the Appendix.

The first nonlinear terms in Eqs. (33) and (34) give account of the self-phase modulation. Notice the symmetry breaking introduced by the waveguide for TM modes. Indeed, the self-action term Γ_m cannot be factorized in the same way as Γ_e (expressions given in Ref. [23]). The $\hat{\chi}^{(3)}_{xyxy}$ and $\hat{\chi}^{(3)}_{xyyx}$ components have then different weights. This leads to different self-focusing powers. The second nonlinear terms in Eqs. (33) and (34) describe the interaction of the two components through the variations of the optical index induced on each component by the other. The last terms in both equations are the four-wave-mixing terms; they enable the energy exchange between the two orthogonal components.

D. Polarization switching

The numerical resolution of Eqs. (33) and (34) shows that the difference between the self-phase modulation constants Γ_e and Γ_m, although very small, is large enough to give account of the experimentally observed commutation of polarization. On the left half of Fig. 4 is represented the evolution of the amplitudes of the two components A_e and A_m, for a hyperbolic secant shaped initial data, corresponding to a soliton, whose polarization is linear, making a 45° angle with the plane of the guide. In this computation, the coefficients have the normalized values valid in a bulk isotropic

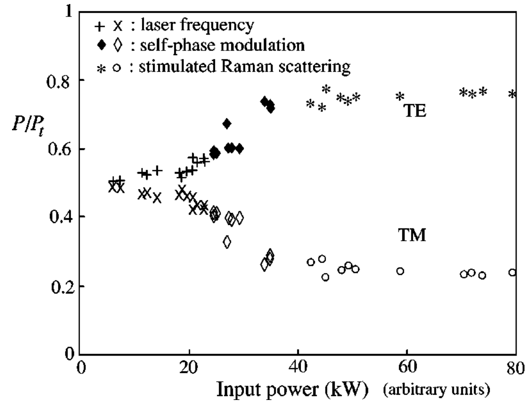

FIG. 5. Relative evolution of the amplitudes of the two polarization components: experimental data to be compared to the theoretical computation of Fig. 6.

medium: $\Gamma_e=\Gamma_m=1$, $g_2=h_2=2/3$, and $g_3=h_3=1/3$. It is seen that the pulse propagates without deformation or variation of its polarization. On the right half of the same figure is represented the evolution of the same amplitude in a slightly different case: the self-phase modulation coefficient of the TM mode has been replaced by $\Gamma_m=0.999$, the remaining being unchanged. It is seen that a commutation, or rather an oscillation, of the polarization occurs. Intentionally, we did not take into account in this computation the variations of the values of the coefficients g_2, h_2, g_3, and h_3 due to the guiding, because the aim of the computation is to show that the computed variation of Γ, despite being small, is sufficient by itself to give account of this experimental result.

Experiments using a waveguide filled with CS_2 have shown a polarization switching, from the TM to the TE polarization, as soon as the incident polarization is not purely TM, i.e., not exactly perpendicular to the plane of the guide, cf. Fig. 5. When the input power is increased, in the first stage where self-phase modulation occurs, the amplitude of the TE component increases while that of the TM one decreases. In the second stage, this phenomenon saturates due to the action of the stimulated Raman scattering, as shown in Fig. 5. The theoretical study of the latter observation is left for further investigation. See Ref. [24] for more details about this experiment. Using the formulas listed in the Appendix, we have computed the accurate values of the nonlinear coefficients of the evolution Eqs. (33) and (34) corresponding to the experimental data. The obtained coefficients are given in Table II. The numerical resolution of Eqs. (33) and (34) using these coefficients has been performed. In order that the simulation can be compared with the experimental data, we computed the amplitudes of the modes at the output for a fixed propagation length and varying input power. This yields Fig. 6. It reproduces the polarization switching with a good agreement with experiment, as can be seen from comparison with the corresponding experimental Fig. 5.

TABLE II. Values of the nonlinear interaction coefficients pertaining to the experimental data.

Γ_e	Γ_m	γ_2	η_2	γ_3	η_3
0.9999	0.9993	0.6681	0.6649	0.3341	0.3324

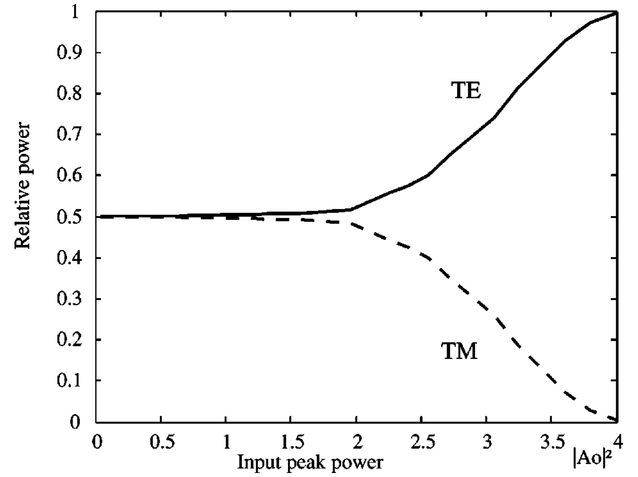

FIG. 6. Relative evolution of the amplitudes of the two polarization components, computed theoretically using the coefficient of Table II.

IV. CONCLUSION

From the Maxwell equations we have derived the general wave propagation equations in a nonlinear waveguide for TE and TM modes. While, regarding its linear part, the computation perfectly agrees with the already known results, the nonlinear coefficient in the nonlinear Schrödinger equation depends on the considered mode and the waveguide geometry. We show in particular that the self-phase modulation coefficients depend on the polarization of the beam, with that of the TM mode inferior to that of the TE one, in agreement with experimental results. Using an expansion formed by a superposition of linear modes, we are able to describe the interaction between a TE and a TM mode, which occurs during the propagation. The description uses two coupled nonlinear Schrödinger equations, whose coefficients have been computed explicitly. We have shown that the small variations of these coefficients that have been computed give account of the polarization instability which had been observed experimentally. Numerical simulations confirmed the dominance of the TE mode upon the TM one.

APPENDIX: NONLINEAR INTERACTION COEFFICIENTS

The coupled nonlinear propagation Eqs. (33) and (34), that describe the coupling between the two polarizations in the waveguide, involve four interaction coefficients g_2, g_3, h_2, and h_3 related to components of the $\chi^{(3)}$ tensor through expressions (35). The expressions of the coefficients γ_2, γ_3, η_2, and η_3 involved in these formulas are given below.

Coefficients g_2 and h_2 express the action of the guide induced by one polarization on the other, and involve γ_2 and η_2, respectively:

$$\gamma_2 = 2\,\mathrm{Re}\left[\frac{3k_{2e}\omega^2}{\gamma_4}\{4\sigma_1\sigma_2 q_m(\mu_1\mu_2 e^{2iq_e a}) + 2q_e^2\sigma_3(\mu_3 e^{2iq_m a}) - (q_e - q_m)q_e\sigma_3(\mu_4\mu_2 e^{2ia(q_m+q_e)}) + (q_m+q_e)q_e\sigma_3(\mu_5\mu_2 e^{2ia(q_e-q_m)}) - 8\sigma_2 q_m\sigma_1[k_{2e} + a(q_e^2 + k_{2e}^2)]\}\right], \tag{A1}$$

$$\eta_2 = 2\mathrm{Re}\left[\frac{3k_{2m}^2(k_{2m}q_m - ik_m^2)q_m}{2\eta_4}(-8q_e\sigma_1\sigma_2(\beta_1 e^{2iq_m a}) + \beta_2 e^{2iq_e a} + \beta_3 e^{2ia(q_m+q_e)} + \beta_4 e^{2ia(q_m-q_e)} + 16\sigma_2\sigma_1 q_m q_e\{k_{2m}\sigma_2(k_m^2 - k_{2m}^2) - a[q_m^2(k_{2m}^4 + k_m^4) + k_{2m}^2\sigma_6]\})\right], \tag{A2}$$

with

$$\beta_1 = \alpha_1\alpha_2^*(-aq_m\alpha_1\alpha_2^* + i\sigma_2 k_{2m}), \tag{A3}$$

$$\begin{aligned}\beta_2 = {} & 4ik_{2m}^4 q_m^3(2k_m^2 q_e^2 - q_m^2\sigma_2) \\ & + 8q_m q_e k_{2m}^3(q_m^2\sigma_2^2 - 2k_m^2 q_e^2\sigma_2) \\ & + 4ik_{2m}^2 q_m^3[2q_e^2(2q_m^2 k_m^2 + q_m^4 - k_m^4) - \sigma_6\sigma_2] \\ & + 8k_{2m}q_e q_m k_m^2(2k_m^2 q_e^2\sigma_2 - q_m^2\sigma_2^2) \\ & + 4iq_m^3 k_m^4(2k_m^2 q_e^2 - q_m^2\sigma_2),\end{aligned} \tag{A4}$$

$$\begin{aligned}\beta_3 = {} & 2ik_{2m}^4 q_m^3(2k_m^2 q_e^2 + q_m q_e\sigma_3 - q_m^2\sigma_2) \\ & + 4k_{2m}^3 q_m\sigma_2(2k_m^2 q_m^2 q_e - 2k_m^2 q_e^3 - q_m q_e^2\sigma_2 + q_m^3\sigma_2) \\ & + 2ik_{2m}^2[(q_m^6 + 9k_m^4 q_m^2 + 3k_m^2 q_m^4 + 3k_m^6)q_m^2 q_e \\ & + q_m^3(k_m^4 + 4q_m^2 k_m^2 + q_m^4)\sigma_2 \\ & - 2(q_m^4 + 3k_m^4 + 2q_m^2 k_m^2)q_m^3 q_e^2 - 4k_m^2 q_e^3\sigma_2^2] \\ & + 4k_{2m}\sigma_2 k_m^2 q_m(2k_m^2 q_e^3 + q_m q_e^2\sigma_2 - 2k_m^2 q_m^2 q_e - q_m^3\sigma_2) \\ & + 2iq_m^3 k_m^4(2k_m^2 q_e^2 + q_m q_e\sigma_3 - q_m^2\sigma_2),\end{aligned} \tag{A5}$$

$$\begin{aligned}\beta_4 = {} & 2ik_{2m}^4 q_m^3(-2k_m^2 q_e^2 + q_m q_e\sigma_3 + q_m^2\sigma_2) \\ & + 4k_{2m}^3 q_m\sigma_2(2k_m^2 q_m^2 q_e - 2k_m^2 q_e^3 + q_m q_e^2\sigma_2 - q_m^3\sigma_2) \\ & + 2ik_{2m}^2[(q_m^6 + 9k_m^4 q_m^2 + 3k_m^2 q_m^4 + 3k_m^6)q_m^2 q_e \\ & - q_m^3(k_m^4 + 4q_m^2 k_m^2 + q_m^4)\sigma_2 \\ & + 2(q_m^4 + 3k_m^4 + 2q_m^2 k_m^2)q_m^3 q_e^2 - 4k_m^2 q_e^3\sigma_2^2] \\ & + 4k_{2m}\sigma_2 k_m^2 q_m(2k_m^2 q_e^3 - q_m q_e^2\sigma_2 - 2k_m^2 q_m^2 q_e + q_m^3\sigma_2) \\ & - 2iq_m^3 k_m^4(2k_m^2 q_e^2 - q_m q_e\sigma_3 - q_m^2\sigma_2)\end{aligned} \tag{A6}$$

and

$$\mu_1 = 1 + k_{2e}a + iq_e a, \tag{A7}$$

$$\mu_3 = q_e^2 - 2q_m^2 + 2ik_{2e}q_m + k_{2e}^2, \tag{A8}$$

$$\mu_4 = k_{2e} + iq_e + 2iq_m, \tag{A9}$$

$$\mu_5 = k_{2e} + iq_e - 2iq_m, \tag{A10}$$

$$\sigma_1 = q_e^2 - q_m^2, \tag{A11}$$

$$\sigma_2 = q_m^2 + k_m^2, \tag{A12}$$

$$\sigma_3 = q_m^2 - k_m^2, \tag{A13}$$

$$\sigma_4 = k_m^4 + k_{2m}^2 q_m^2, \tag{A14}$$

$$\sigma_5 = k_{2m}^2 - q_m^2, \tag{A15}$$

$$\sigma_6 = k_m^4 + q_m^4, \tag{A16}$$

$$\alpha_1 = ik_m^2 + k_{2m}q_m, \tag{A17}$$

$$\alpha_2 = q_m + ik_{2m}. \tag{A18}$$

Coefficients g_3 and h_3 describe the energy exchange between the TE and TM components, as usually do four wave mixing terms. These terms depend only on the $\hat{\chi}^{(3)}_{xyyx}$ component of the nonlinear susceptibility tensor. The waveguide action is represented by the coefficients γ_3 and η_3,

$$\begin{aligned}\gamma_3 = 2\,\mathrm{Re}\Big[\frac{3k_{2e}\omega^2}{2\gamma_4}\{ & -4\sigma_1\sigma_3 q_m(\mu_1\mu_2 e^{2iq_e a}) \\ & - 2q_e^2\sigma_2(\mu_3 e^{2iq_m a}) + (q_e - q_m)q_e\sigma_2(\mu_4\mu_2 e^{2ia(q_m+q_e)}) \\ & - (q_m + q_e)q_e\sigma_2(\mu_5\mu_2 e^{2ia(q_e-q_m)}) \\ & + 8\sigma_3 q_m\sigma_1[k_{2e} + a(q_e^2 + k_{2e}^2)]\}\Big],\end{aligned} \tag{A19}$$

$$\begin{aligned}\eta_3 = 2\,\mathrm{Re}\Big[\frac{3k_{2m}^2(k_{2m}q_m - ik_m^2)q_m}{4\eta_4}(& 8iq_e\sigma_1\sigma_2(\delta_1 e^{2iq_m a}) \\ & + \delta_2 e^{2iq_e a} + \delta_3 e^{2ia(q_m+q_e)} + \delta_4 e^{2ia(q_m-q_e)} \\ & + 16\sigma_4\sigma_1 q_m q_e\{k_{2m}\sigma_2(k_{2m}^2 - k_m^2) \\ & + a[q_m^2(k_{2m}^4 + k_m^4) + k_{2m}^2\sigma_6]\})\Big],\end{aligned} \tag{A20}$$

with

$$\delta_1=\alpha_1\alpha_2^*(iaq_m\alpha_1\alpha_2^*+k_{2m}\sigma_2), \tag{A21}$$

$$\begin{aligned}\delta_2=&4ik_{2m}^4q_m^3(2k_m^2q_e^2+q_m^2\sigma_3)\\&+8\sigma_2q_mq_ek_{2m}^3(-q_m^2\sigma_3-2k_m^2q_e^2)\\&-4ik_{2m}^2q_m^3[2q_e^2(2k_m^2q_m^2+q_m^4+3k_m^4)-\sigma_6\sigma_3]\\&+8q_2q_eq_mk_m^2\sigma_2(2k_m^2q_e^2+q_m^2\sigma_3)\\&+4iq_m^3k_m^4(2k_m^2q_e^2+q_m^2\sigma_3),\end{aligned} \tag{A22}$$

$$\begin{aligned}\delta_3=&2ik_{2m}^4q_m^3(2k_m^2q_e^2-q_mq_e\sigma_2+q_m^2\sigma_3)\\&+4k_{2m}^3q_m\sigma_2(2k_m^2q_m^2q_e-2k_m^2q_e^3+q_mq_e^2\sigma_3-q_m^3\sigma_3)\\&+2ik_{2m}^2[-(q_m^4+4k_m^2q_m^2+k_m^4)q_m^3\sigma_3\\&+(-q_m^4+4k_m^2q_m^2+3k_m^4)\sigma_2q_m^2q_e\\&-2q_m^3q_e^2(-q_m^4-2k_m^2q_m^2+k_m^4)-4k_m^2q_e^3\sigma_2^2]\\&+4k_{2m}\sigma_2k_m^2q_m(2k_m^2q_e^3-q_mq_e^2\sigma_3-2k_m^2q_m^2q_e+q_m^3\sigma_3)\\&+2iq_m^3k_m^4(2k_m^2q_e^2-q_mq_e\sigma_2+q_m^2\sigma_3),\end{aligned} \tag{A23}$$

$$\begin{aligned}\delta_4=&2ik_{2m}^4q_m^3(-2k_m^2q_e^2-q_mq_e\sigma_2-q_m^2\sigma_3)\\&+4k_{2m}^3q_m\sigma_2(2k_m^2q_m^2q_e-2k_m^2q_e^3-q_mq_e^2\sigma_3+q_m^3\sigma_3)\\&+2ik_{2m}^2[(-q_m^4+4k_m^2q_m^2+3k_m^4)\sigma_2q_m^2q_e\\&+(q_m^4+4k_m^2q_m^2+k_m^4)q_m^3\sigma_3\\&+2q_m^3q_e^2(-q_m^4-2k_m^2q_m^2+k_m^4)-4k_m^2q_e^3\sigma_2^2]\\&+4k_{2m}\sigma_2k_m^2q_m(2k_m^2q_e^3+q_mq_e^2\sigma_3-2k_m^2q_m^2q_e-q_m^3\sigma_3)\\&-2iq_m^3k_m^4(2k_m^2q_e^2+q_mq_e\sigma_2+q_m^2\sigma_3).\end{aligned} \tag{A24}$$

The factors γ_4 and η_4 in the denominators of expressions (A1), (A2), (A19), (A20) have the following expressions:

$$\gamma_4=[(1+k_{2e}a)\sigma_1q_mk_m^2(3\mu_2^2e^{2iq_ea}+\mu_2^{*2}e^{-2iq_ea})], \tag{A25}$$

$$\begin{aligned}\eta_4=&2k_mn^2q_e\sigma_1\alpha_1\alpha_2^{*2}[(3k_{2m}^2q_ma\sigma_3+3q_mk_{2m}^3\sigma_4+ik_m^4\sigma_5\\&+3k_{2m}q_mk_m^2\sigma_2)e^{2iq_ma}+i\alpha_1^{*2}\alpha_2^2(aq_mk_{2m}^2\alpha_1^*+k_{2m}^2\sigma_4\\&+q_mk_m^2\alpha_2^*)e^{-2iq_ma}].\end{aligned} \tag{A26}$$

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